AD/A-006 032

SHORT-TERM SPOT SIZE AND BEAM WANDER IN A TURBULENT MEDIUM

Ronald L. Fante, et al

Air Force Cambridge Research Laboratories
Hanscom Air Force Base, Massachusetts

27 November 1974

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFCRL-TR-74-0595	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD/A-006632
4. TITLE (and Subtitle) SHORT-TERM SPOT SIZE AND BEAM WANDER IN A TURBULENT MEDIUM		5. TYPE OF REPORT & PERIOD COVERED Scientific. Interim.
		6. PERFORMING ORG. REPORT NUMBER PSRP No. 616
7. AUTHOR(s) Ronald L. Fante and Richard L. Taylor		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Cambridge Research Laboratories (LZP) Hanscom AFB Massachusetts 01731		10. PROGRAM ELEMENT PROJECT TASK AREA & WORK UNIT NUMBERS 21530201 61102F 681305
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Cambridge Research Laboratories (LZP) Hanscom AFB Massachusetts 01731		12. REPORT DATE
		13. NUMBER OF PAGES 15
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES TECH. OTHER		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Turbulent propagation Atmospheric optics Beam wander		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) We have calculated the short-term beam spread and beam wander of a laser beam propagating in a turbulent medium. These results have been used to calculate the probability that, due to beam wander, the beam will fail to hit a receiver aperture.		

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1. INTRODUCTION

One difficulty in using laser systems for communication through the earth's atmosphere is that there is a possibility that the beam may wander from position to position, and may therefore completely miss the receiver in some instances. Since the spot dances from position to position in times of order 1 millisecond, this would not be a problem for systems using pulses much longer than 1 millisecond, but would be an important consideration for systems using pulses short compared with a millisecond. In this report we shall therefore study the nature of the beam wander and derive results appropriate for application to laser communications systems.

2. DERIVATION OF BEAM-WANDER FORMULA

It is well understood from purely physical considerations that eddies which are large compared with the diameter of a laser beam tend to deflect the beam, while those smaller than the beam tend to broaden the beam but do not deflect it significantly. Let us consider the nature of the received spot on an aperture in a turbulent medium. If we look over very short times we see a broadened spot; as we look for longer times we see that the spot dances from position to position. Therefore if we average the received intensity over very long times, the total broadened spot would consist of two components: actual short-term beam

(Received for publication 25 November 1974)

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broadening due to scatter by the small eddies, and beam wander due to the effect of the large eddies. The temporal history of the received spot would be as indicated below* in Figure 1.

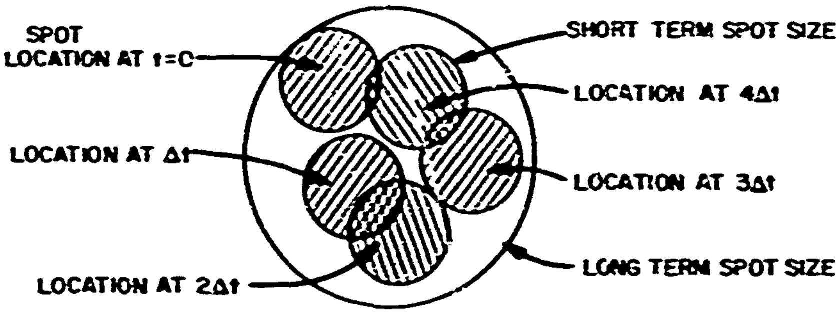

Figure 1. Time History of the Wander of a Laser Beam in a Turbulent Medium

In the above figure, $\Delta t = D/v$ where D is the beam diameter and v is the transverse component of the wind. The long-term spot is what is obtained by averaging over times $T \gg \Delta t$.

The long- and short-term beam irradiance can be obtained from the modified Huygens-Fresnel principle. The short-term irradiance[1,2,3] is

$$< I(\rho) >_{ST} = \left(\frac{k_o}{2\pi L}\right)^2 \iint d^2\rho' \, M_{ST}(\rho', L) \exp\left\{-i \frac{k_o}{L} \underline{\rho} \cdot \underline{\rho}'\right\}$$

$$\cdot \iint d^2\xi \; e_o\left(\xi + \frac{\rho'}{2}\right) e_o^*\left(\xi - \frac{\rho'}{2}\right) \exp\left\{i \frac{k_o}{L} \xi \cdot \rho'\right\}. \qquad (1)$$

*The above argument holds for $k_o^{7/6} C_n^2 L^{11/6} < 1$, where k_o is the signal wavenumber, C_n^2 is the index of refraction structure constant, and L is the path length. For $k_o^{7/6} C_n^2 L^{11/6} \gg 1$, we expect that the beam will be broken up into multiple patches with negligible wander of the beam centroid.

1. Kon, A. (1970) Focusing of light in a turbulent medium, Radiophysics and Quantum Electronics 13:43-50.

2. Fried, D.L. (1966) Optical resolution through a randomly inhomogeneous medium for very long and very short exposures, J. Opt. Soc. Am. 56:1372-1379.

3. Yura, H. (1973) Short-term average optical beam spread in a turbulent medium, J. Opt. Soc. Am. 63:567-572.

where e_o is the aperture field of the transmitter, L is the path length in turbulence, k_o is the wavenumber, and M_{ST} is the short-term modulation transfer function given by[4] (assuming $D > \ell_o$ where ℓ_o is the inner scale of the turbulence)

$$M_{ST}(\rho, L) = \exp\left\{\frac{-0.13\pi^2 k_o^2 \rho^{5/3}}{L^{5/3}} \int_0^L dz'(z')^{5/3} C_n^2(z') \int_{\gamma\left(\frac{\rho}{D}\right)}^{\infty} d\xi\, \xi^{-8/3}\left[1 - J_o(\xi)\right]\right\} \quad (2)$$

where γ is a number of order unity and C_n^2 is the index of refraction structure constant. In Eq. (2) the integral on ξ from $\gamma(\rho/D)$ to ∞ indicates that the effect on the MTF of eddies larger than D is excluded. For $\gamma(\rho/D) \leq 1$, Eq. (2) can be approximated by (assuming also that $D < L_o$ where L_o is the outer scale size of the turbulence)

$$M_{ST} \sim \exp\left\{-\left(\frac{\rho}{\rho_o}\right)^{5/3}\left[1 - 0.67\left(\frac{\gamma\rho}{D}\right)^{1/3} + 0.005\left(\frac{\gamma\rho}{D}\right)^{7/3} - 0.88 \times 10^{-4}\left(\frac{\gamma\rho}{D}\right)^{13/3}\right]\right\}. \quad (3a)$$

For $\gamma(\rho/D) \gg 1$, we can approximate M_{ST} by

$$M_{ST} \simeq \exp\left\{-\frac{1.43}{\gamma^{5/3}}\left(\frac{D}{\rho_o}\right)^{5/3}\right\}. \quad (3b)$$

where

$$\rho_o = \left[1.46\, k_o^2\, L^{-5/3} \int_0^L dz'\,(z')^{5/3}\, C_n^2(z')\right]^{-3/5}. \quad (4)$$

For $\gamma\left(\frac{\rho}{D}\right)$ of order 1, we have not been able to derive any approximate results for M_{ST}, and the full expression of Eq. (2) must be used.

The long-term averaged irradiance $\langle I \rangle_{LT}$ is given by Eq. (1) with M_{ST} replaced by the long-term modulation transfer function M_{LT}. M_{LT} is given by Eq. (2) with $\gamma = 0$, and is

$$M_{LT} = \exp\left[-\left(\frac{\rho}{\rho_o}\right)^{5/3}\right]. \quad (5)$$

4. Clifford, S., Ochs, G., and Lawrence, R. (1974) Saturation of optical scintillation by strong turbulence. J. Opt. Soc. Am. 64:148-154.

In general, the expressions for the long- and short-term irradiance must be evaluated numerically, as we have done in Section 3. However, there are limiting cases where approximate analytical results can be obtained.

(a) $\gamma\rho_o/D \gg 1$. In this case $M_{ST} \simeq M_{LT} \simeq 1$, and we get

$$<I>_{ST} \simeq <I>_{LT} \simeq 2\beta^2 \int_0^\infty y\,dy\,J_o(\alpha y)\,e^{-y^2(1+\beta^2)} \tag{6}$$

$$= I_{VACUUM}$$

where we have assumed an aperture distribution

$$e_o = \exp\left[-\frac{2\rho^2}{D^2}\right]. \tag{7}$$

and have defined

$$\alpha = \frac{k_o D\rho}{L}, \qquad \beta^2 = \frac{k_o^2 D^4}{16 L^2}.$$

Therefore if the coherence length ρ_o is much greater than the aperture diameter, the long- and short-term averaged irradiances at the receiver are the same as would be present at the receiver if the turbulent medium were replaced by vacuum.

(b) $\gamma\rho_o/D \ll 1$. In this case the short- and long-term averaged irradiances can be approximated by

$$<I(\rho)>_{ST} = 2\beta^2 \int_0^\infty y\,dy\,J_o(\alpha y)\,\exp\left\{-\left(\frac{Dy}{\rho_o}\right)^{5/3}\left[1-0.57\,(\gamma y)^{1/3}\right]-y^2(1+\beta^2)\right\}. \tag{8}$$

and

$$<I(\rho)>_{LT} = 2\beta^2 \int_0^\infty y\,dy\,J_o(\alpha y)\,\exp\left\{-\left(\frac{Dy}{\rho_o}\right)^{5/3} - y^2(1+\beta^2)\right\}. \tag{9}$$

Equations (8) and (9) are readily evaluated numerically. However, it is possible to estimate the radius of the beam by approximating

$$\exp\left\{-\left(\frac{Dy}{\rho_o}\right)^{5/3}\left[1 - 0.67\,(\gamma y)^{1/3}\right]\right\}$$

by $\exp(-y^2/y_o^2)$, where y_o satisfies

$$\left(\frac{Dy_o}{\rho_o}\right)^{5/3}\left[1-0.67\,(\gamma y_o)^{1/3}\right]=1\,. \tag{10}$$

If we use this approximation we have

$$\langle I(\rho)\rangle_{ST}=\frac{\beta^2}{1+\beta^2+y_o^{-2}}\exp\left\{-\frac{\alpha^2}{4\left(1+\beta^2+y_o^{-2}\right)}\right\}. \tag{11}$$

From Eq. (10) we see that the mean square beam radius is

$$\sigma_{ST}^2=4\left(1+\beta^2+y_o^{-2}\right)\left(\frac{L}{k_oD}\right)^2$$

$$=\frac{4L^2}{k_o^2D^2}+\frac{D^2}{4}+\frac{4L^2}{k_o^2D^2y_o^2}\,. \tag{12}$$

If we solve Eq. (10) for y_o, we find (for $\gamma\rho_o/D \ll 1$)

$$y_o\simeq\frac{\rho_o}{D}\left[1-0.67\left(\frac{\gamma\rho_o}{D}\right)^{1/3}\right]^{-3/5}. \tag{13}$$

so that

$$\sigma_{ST}^2\simeq\frac{4L^2}{k_o^2D^2}+\frac{D^2}{4}+\frac{4L^2}{k_o^2\rho_o^2}\left[1-0.67\left(\frac{\gamma\rho_o}{D}\right)^{1/3}\right]^{6/5}. \tag{14}$$

Equation (14) gives the mean square short-term beam radius. The long-term mean square beam radius is readily obtained from Eq. (14) by setting $\gamma=0$. We get

$$\sigma_{LT}^2=\frac{4L^2}{k_o^2D^2}+\frac{D^2}{4}+\frac{4L^2}{k_o^2\rho_o^2}\,. \tag{15}$$

From Eqs. (14) and (15) it is now possible to calculate the mean square beam wander σ_w, since the mean square beam wander is related to σ_{LT} and σ_{ST} as

$$\sigma_{LT}^2 = \sigma_{ST}^2 + \sigma_W^2 . \tag{16}$$

If we use Eqs. (14) and (15) in (16), we get

$$\sigma_W^2 = \frac{4L^2}{k_o^2 \rho_o^2} \left\{ 1 - \left[1 - 0.67 \left(\frac{\gamma \rho_o}{D} \right)^{1/3} \right]^{6/5} \right\} . \tag{17}$$

Since we have been assuming that $(\gamma \rho_o / D) \ll 1$, we can approximate Eq. (17) further by

$$\sigma_W^2 \simeq \frac{3.24\, \gamma^{1/3} L^2}{k_o^2 D^{1/3} \rho_o^{5/3}} = \frac{4.70\, \gamma^{1/3} L^2}{D^{1/3} L^{5/3}} \int_0^L dz' \, C_n^2(z') \, (z')^{5/3} . \tag{18}$$

Now $\sigma_W^2 = L^2 \langle \phi^2 \rangle$, where $\langle \phi^2 \rangle$ is the mean square angle of wander of the beam. We therefore have for $\gamma \rho_o / D \ll 1$:

$$\langle \phi^2 \rangle = \frac{4.70\, \gamma^{1/3}}{D^{1/3} L^{5/3}} \int_0^L dz' \, C_n^2(z') \, (z')^{5/3} . \tag{19}$$

We have not yet specified $\gamma^{1/3}$, but estimates obtained by comparison with data appear to indicate that $0.9 < \gamma^{1/3} < 1.7$. We shall choose $\gamma^{1/3} = 1.0$, since this gives a reasonable fit with the data of Dowling and Livingston,[5] and with the theoretical results of Fried.[2]

3. NUMERICAL RESULTS

Equations (14) and (15) give the short- and long-term beam spread for $\gamma \rho_o / D \ll 1$. However, we will also often be interested in the case when $\gamma \rho_o / D \sim 1$. In this case the long- and short-term irradiances must be evaluated numerically. If we assume the aperture field is given by Eq. (7), then we must evaluate

$$\langle I(\rho) \rangle_{ST} = 2\beta^2 \int_0^\infty y\, dy\, J_o(\alpha y) \exp\left\{ -y^2 (1+\beta^2) - f_s(y) \right\} . \tag{20}$$

5. Dowling, J. and Livingston, P. (1973) Behavior of focussed beams in atmospheric turbulence: Measurements and comments on the theory. J. Opt. Soc. Am. 63:846-858.

$$\langle I(\rho)\rangle_{LT} = 2\beta^2 \int_0^\infty y\,dy\,J_o(\alpha y)\exp\left\{-y^2(1+\beta^2) - \left(\frac{Dy}{\rho_o}\right)^{5/3}\right\}, \tag{21}$$

where

$$f_s(y) = y^{5/3}\left(\frac{D}{\rho_o}\right)^{5/3}\left\{1 - 0.91\int_0^{\gamma y} d\xi\,\xi^{-8/3}\left[1 - J_o(\xi)\right]\right\}. \tag{22}$$

Equations (20) and (21) have been evaluated numerically to determine the long-term and short-term beamwidth as a function of $\gamma\rho_o/D$, for various values of β^2 when $\gamma = 1$. These results are shown in Figure 2. From this figure we can readily determine the mean square beam wander since

$$\begin{aligned}
\sigma_W^2 &= \sigma_{LT}^2\left(1 - \frac{\sigma_{ST}^2}{\sigma_{LT}^2}\right) \\
&= \left[\frac{4L^2}{k_o^2 D^2}\left(1 + \frac{D^2}{\rho_o^2}\right) + \frac{D^2}{4}\right]\left(1 - \frac{\sigma_{ST}^2}{\sigma_{LT}^2}\right) \\
&= \frac{D^2}{4}\left[\frac{1}{\beta^2}\left(1 + \frac{D^2}{\rho_o^2}\right) + 1\right]\left(1 - \frac{\sigma_{ST}^2}{\sigma_{LT}^2}\right).
\end{aligned} \tag{23}$$

and σ_{ST}/σ_{LT} is given in Figure 2.

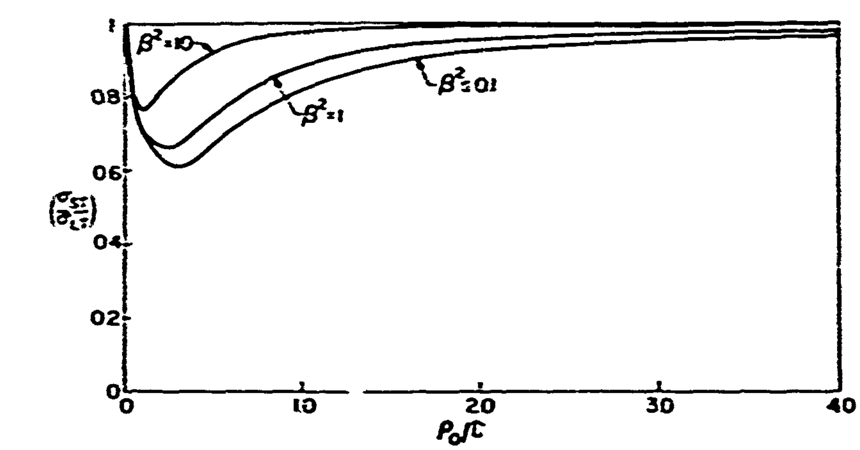

Figure 2. Ratio of the Short-Term to Long-Term Spot Size of a Laser Beam in a Turbulent Medium

Since the beam wander is a gaussian random variable,[5] we can now calculate the probability that the beam will miss a point aperture. We have for the probability of the beam wander

$$P(\rho_W) = \frac{1}{2\pi\sigma_W^2} \exp\left\{-\frac{\rho_W^2}{2\sigma_W^2}\right\} . \tag{24}$$

The probability that the beam will miss (by miss we mean that the amplitude at the receiver will be reduced by more than e^{-1}) a point aperture is equal to the probability that the beam wander exceeds the short-term beam spread. That is

$$P_{MISS} = 2\pi \int_{\sigma_{ST}}^{\infty} P(\rho_W)\, \rho_W \, d\rho_W$$

$$= \exp\left\{-\frac{1}{2}\left(\frac{\sigma_{ST}}{\sigma_{LT}}\right)^2\right\} . \tag{25}$$

Upon using Eq. (23) in (25) we have

$$P_{MISS} = \exp\left\{-\frac{\frac{1}{2}\left(\frac{\sigma_{ST}}{\sigma_{LT}}\right)^2}{1-\left(\frac{\sigma_{ST}}{\sigma_{LT}}\right)^2}\right\} . \tag{26}$$

This result is plotted in Figure 3. For the case of a receiving aperture of radius A_R, we replace σ_{ST} by $A_R + \sigma_{ST}$ in Eq. (25). In the next section we will do an example to illustrate the use of Figures 2 and 3 for atmospheric propagation.

4. APPLICATION TO THE EARTH'S ATMOSPHERE

An important application of our results is to a laser communications link between the earth and an orbiting satellite. Let us suppose the satellite is at an altitude h_s and at an angle θ relative to the normal to the earth at the transmitter, as shown in Figure 4. For this case we may write

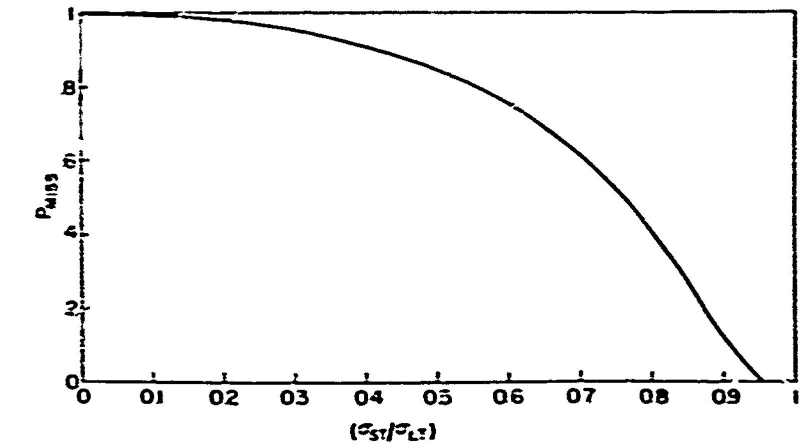

Figure 3. Probability That a Laser Beam Will Miss a Point Receiving Aperture, as a Function of the Ratio of the Short-Term to Long-Term Spot Size

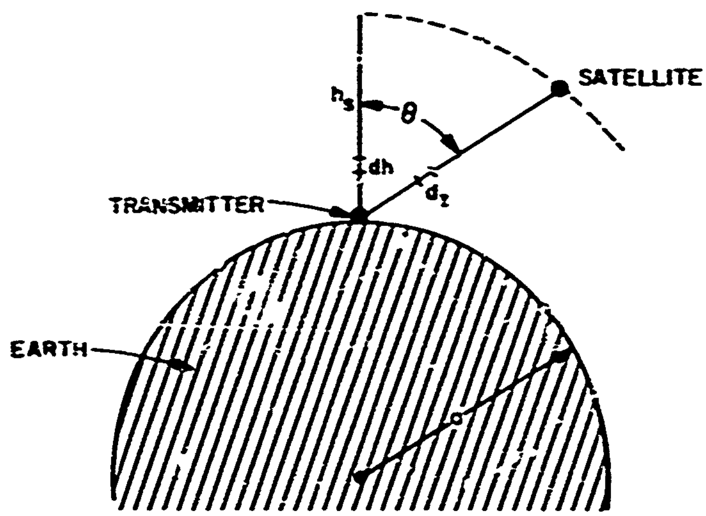

Figure 4. Model of Earth-to-Satellite Laser Beam Transmission

$$dz = \frac{(h + a)\, dh}{\sqrt{h^2 + 2ah + a^2 \cos^2\theta}} \quad . \tag{27}$$

where a is the radius of the earth. If we assume $h_s \ll a$ and

$$\theta < \cos^{-1}\left[\left(\frac{2h_s}{a}\right)^{1/2}\right] . \tag{28}$$

we can approximate Eq. (27) by

$$dz = \sec\theta \, dh . \tag{29}$$

and we may therefore evaluate ρ_o. for the earth-to-satellite path as

$$\rho_o^{-5/3} \doteq \frac{1.46 \, k_o^2 \sec\theta}{h_s^{5/3}} \int_0^{h_s} dh \, C_n^2(h) \, (h)^{5/3} . \tag{30}$$

A commonly used distribution for C_n^2 is (31)

$$C_n^2 = C_o^2 \, h^{-\nu} \exp\left[-h/h_o\right] .$$

so that for $h_s \gg h_o$

$$\rho_o^{-5/3} = 1.46 \, k_o^2 \sec\theta \, \Gamma\left(\frac{8}{3} - \nu\right) C_o^2 \, h_o^{8/3-\nu} \, h_s^{-5/3} . \tag{32}$$

where Γ is the gamma function. If we take $C_o^2 = 10^{-13}$, $\nu = 1/3$ and $h_o = 1000$ meters, we get

$$\rho_o = h_s \left[8.05 \times 10^{-9} \, k_o^2 \sec\theta\right]^{-3/5} \text{ meters.} \tag{33}$$

For $\theta = 30°$ and $k_o = 0.99 \times 10^7 \text{ m}^{-1}$ (0.633 μm light), we have

$$\rho_o = 2.63 \times 10^{-4} \, h_s \text{ meters.} \tag{34}$$

Now suppose the transmitter diameter D is 1 m and the satellite is at an altitude h_s of 200 km. Then $\rho_o/D = 52.5$ and $\beta^2 = 10^{-3}$. Consequently, from Figure 2 we have $\sigma_{ST}/\sigma_{LT} = 1$ and from Figure 3 we see that the probability that the beam (over times of order D/v) will miss a point receiver on the satellite is $P_{MISS} \approx 0$. Of course as $\theta \rightarrow 90°$, P_{MISS} will no longer be nearly equal to zero.